BUSINESS MATHEMATICS

COURSE CODE 1429

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ALLAMA IQBAL OPEN UNIVERSITY

Level:	BS/B.Ed./Associate Degree	Semester:	Spring, 2024
Course & Code	Business Mathematics - 1429	Maximum Marks:	100
Time Allowed:	03 Hours	Pass Marks:	40

Note: ATTEMPT ANY FIVE QUESTIONS.

Q. No.	Questions	Marks
Q.No.1	An urn contains 5 white and 3 black balls. Two balls are drawn at random without replacement. If X denotes the number of white balls, then find the probability distribution of X.	20
Q.No.2	a) Solve: $\lvert 7x - 12\rvert = \lvert 4 - 3x\rvert$. b) Find the following limit: $\lim_{x \to 5}\left[\left(\frac{x+3}{x+6}\right)(x^2 - 12)\right]$.	10+10
Q.No.3	Construct the discrete probability distribution which corresponds to the experiment of the single roll of a pair of dice. Assume equal likelihood of occurrence of each side of a die and let the random variable X equal the sum of the dots which appear on the pair.	20
Q.No.4	The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (Stated in dollars) and the number of units produced x is $P = -0.01x^2 + 5{,}000x - 25{,}000$ (i) Determine the number of units x which will result in maximum profit. (ii) What is the expected maximum profit?	10+10
Q.No.5	Find $f'(x)$, $f''(x)$, $f'(1)$ and $f''(1)$ when $f(x) = \frac{2x}{x^2+1}$.	20
Q.No.6	Find A^{-1} and show that $AA^{-1} = I$, if $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{bmatrix}$.	20
Q.No.7	a) Solve the inequality: $x^2 + 4x - 12 \leq 0$. b) Solve graphically and check your answer algebraically: $-x + 3y = 2; \quad 4x - 12 = -8$.	10+10
Q.No.8	a) Determine the location and values of the absolute maximum and absolute minimum for the given function: $f(x) = (-6x + 7)^4; \quad 0 \leq x \leq 3$. b) Sketch the graph of the function: $f(x) = e^{-2x} - 4x$.	10+10

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Level	BA/AD/BS/B.Ed	Semester	Autumn 2023	
Subject/Course Code	Business Mathematics (1429)	Maximum Marks	100	
Maximum Time	03 Hours	Pass Marks	BA	40
Medium of Instructions	English		AD/BS/B.Ed	50

NOTE: <u>Attempt any FIVE questions</u>

Q.No.	Questions	Marks
1.	a) A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day (assume that an over dosage of either drug is harmless). Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. How many grams of substances M and N can be mixed to meet the minimum daily requirements? b) Solve $\lvert 7x - 12\rvert = \lvert 4 - 3x\rvert$	10+10
2	The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (Stated in dollars) and the number of units produced x is $P = -0.01x^2 + 5{,}000x - 25{,}000$ (i) Determine the number of units x which will result in maximum profit. (ii) What is the expected maximum profit?	20
3	Solve graphically and check your answer algebraically. $-x + 3y = 2$ $4x - 12 = -8$	20
4	Find A^{-1} and show that $AA^{-1} = I$, if $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{bmatrix}$	20
5	Determine the location and values of the absolute maximum and absolute minimum for the given function: $f(x) = (-6x + 7)^4$, where $0 \le x \le 3$	20
6	Find $f'(x)$, $f''(x)$, $f'''(x)$, $f'(1)$, $f''(1)$, $f''(5)$ and $f'''(5)$ when $f(x) = \frac{2x}{x^2+1}$	20
7	For a given function $f(x) = x^4 + x^2 + 10$ (i) find those open intervals over which the function is concave up, concave down. (ii) find all points of inflection.	10+10
8	Define critical point, point of inflection. Find all relative extrema, using First Derivative Test. Sketch a graph of the function $f(x) = \frac{x^2}{x-1}$	20

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Level	BA/AD/BS/B.Ed	Semester	Spring 2023	
Subject/Course Code	Business Mathematics (1429)	Maximum Marks	100	
Maximum Time	03 Hours			
Medium of Instructions	English	Pass Marks	BA	40
			AD/BS/B.Ed	50

NOTE: Attempt any FIVE questions

Q.No.	Questions	Marks
1.	a) A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day (assume that an over dosage of either drug is harmless). Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. How many grams of substances M and N can be mixed to meet the minimum daily requirements? b) Solve: $\|7x - 12\| = \|4 - 3x\|$	10+10
2	The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (Stated in dollars) and the number of units produced x is $P = -0.01x^2 + 5{,}000x - 25{,}000$ (i) Determine the number of units x which will result in maximum profit. (ii) What is the expected maximum profit?	20
3	a) Solve the following second degree inequality $x^2 + 4x - 12 \leq 0$ b) Solve graphically and check your answer algebraically. $-x + 3y = 2$ $4x - 12 = -8$	10+10
4	Find A^{-1} and show that $AA^{-1} = I$, if $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{bmatrix}$	20

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Level:	B.Com	Semester:	Spring, 2023
Course & Code	Business Mathematics (1429)	Maximum Marks:	100
Time Allowed:	03 Hours	Pass Marks:	40/50

Note: ATTEMPT FIVE QUESTIONS. ALL CARRY EQUAL MARKS.

Q. No.	Questions	Marks
Q.No.1	Define Critical Point. Determine the interval over which f is i) increasing ii) decreasing iii) neither increasing nor decreasing iv) concave up v) concave down. If $$f(x) = \frac{2x^3}{3} - 5x^2 - \frac{1}{x} - 3$$	20
Q.No.2	(a) Sketch the plane: $3x_1 = 9$ (b) $C = \frac{5}{9}F - \frac{160}{9}$ is an equation relating temperature in Celsius units to temperature measured on the Fahrenheit scale. Let C = degrees Celsius and F = degrees Fahrenheit; assume the equation is graphed with C measured on the vertical axis i. Identify the slope and y-intercept ii. Interpret the meaning of the slope and y intercept for purposes of converting from Fahrenheit to Celsius temperature. iii. Solve the equation for F and rework parts i and ii if F is plotted on vertical axis.	8+12=20
Q.No.3	(a) The sample space of an experiment consists of five simple events E_1, E_2, E_3, E_4 *and* E_5. These events are mutually exclusive. The probabilities of occurrence of these events are $P(E_1) = .20,\ P(E_2) = .15, P(E_3).25, P(E_4) = .30, P(E_5) = .10$. Several compound events can be defined for this experiment. They are $F = \{E_1, E_2, E_3\},\ G = \{E_1, E_3, E_5\}, H = \{E_4, E_5\}$. Determine (i) $P(G')$ (ii) $P(F \cup G)$ (iii) $P(G \cup H)$ (iv) $P(F \cap H)$ (v) $P(F \cap G)$ (b) Solve the following second degree inequality $\boldsymbol{x + 4x - 12 \le 0}$	12+8=20
Q.No.4	(a) Determine the inverse of $\begin{pmatrix} 11 & 4 & 7 \\ -7 & -3 & 1 \\ 5 & -6 & 9 \end{pmatrix}$ by using the matrix of cofactor approach. (b) Sketch the graph of each of the following linear functions. i) Graph passes through the point (-3,-2) with slope of -1. ii) Graph passes through the point (2, 4) with slope of 0.	12+8=20

	iii) Graph passes through the point (5, 0) with undefined slope.	
Q.No.5	In a certain marketplace the demand and supply functions for a commodity are as follows: $$D: p = 100 - 5q$$ $$S: p = 20 + 4q$$ i) What are the initial equilibrium price and quantity? ii) Assume an imaginative advertising campaign shifts the demand function two places to the right. Sketch the initial demand and supply functions. Sketch the new demand function. Find the new equilibrium price and quantity. iii) Assume that a tax of $1 per unit is levied on the seller. What will be the effect on the supply function? Depict the situation graphically. Determine the new equilibrium price and quantity.	20
Q.No.6	In a survey of 410 salespersons and 350 construction workers, it is found that 164 of the salespersons and 196 of the construction workers were overweight. If a person is selected at random from the group, what is the probability that: i) This person is overweight? ii) This person is a salesperson, given that the person is overweight? iii) This person is overweight, given that the person is a salesperson? iv) This person is a construction worker, given that the person is not overweight? This person is not overweight, given that the person is a construction worker?	20
Q.No.7	(a) Find $f_x, f_y, f_z, f_{yx}, f_{xz}, f_{zy}, f_y(2,-1,3), f_{yz}(-1,1,0)$. if $$f(x,y,z) = \frac{2x^2+xy}{yz-2}$$ (b) Suppose $z = f(x,y)$ describes the cost to build a certain structure, where x represents the labor costs and y represents the cost of materials. Describe what f_x *and* f_y represent.	14+6=20
Q.No.8	(a) A ball throw upward from the roof of a building which is 750 feet high will be at a height of h feet after t seconds as described by the function. $$h = f(t) = -16t^2 + 50t + 750$$ (i) What is the height of a ball after 5 seconds? (ii) What is the velocity of the ball after 5 seconds (iii) What is the acceleration of the ball at $t = 0$? At $= 5$? (b) Find a, b, c and d so that $$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$	10+10=20

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Level:	BS/B.Ed./Associate Degree	Semester:	Aut, 2022
Course & Code	Business Mathematics - 1429	Maximum Marks:	100
Time Allowed:	03 Hours	Pass Marks:	50

Note: ATTEMPT ANY FIVE QUESTIONS.

Q. No.	Questions	Marks
Q.No.1	(i) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected from each group. Find the probability that the three selected consist of 1 girl and 2 boys. (ii) Solve: $\lvert 7x - 12\rvert = \lvert 4 - 3x\rvert$	20
Q.No.2	Determine the solution the system of equation. $x_1 + 3x_2 + 3x_3 = 15$ $-2x_2 - 3x_3 = 12$ $-x_1 - 2x_2 - x_3 = 10$	20
Q.No.3	Find a, b, c and d so that $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$	20
Q.No.4	(a) Construct the discrete probability distribution which corresponds to the experiment of the single roll of a pair of dice. Assume equal likelihood of occurrence of each side of a die and let the random variable X equal the sum of the dots which appear on the pair. (b) Find the following limit: $\lim_{x\to 3} \sqrt{x^3 - 27}$.	10+10
Q.No.5	For a given function f(x): a) find those open intervals over which the function is concave up, concave down. b) find all points of inflection. $f(x) = x\text{^}4 + x\text{^}2 + 10$	10+10
Q.No.6	a) Identify the location of any inflection points for the following functions.	

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Level:	BS/B.Ed./Associate Degree	Semester:	Spring 2023
Course & Code	Business Mathematics - 1429	Maximum Marks:	100
Time Allowed:	03 Hours	Pass Marks:	50

Note: ATTEMPT ANY FIVE QUESTIONS.

Q. No.	Questions	Marks
Q.No.1	a) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected from each group. Find the probability that the three selected consist of 1 girl and 2 boys. b) Solve: $\lvert 7x - 12\rvert = \lvert 4 - 3x\rvert$	10+10
Q.No.2	a) Let X be a random variable giving the number of aces in a random draw of 4 cards from an ordinary deck of 52 cards. Construct a table showing the probability distribution of X. b) An urn contains 5 white and 3 black balls. Two balls are drawn at random without replacement. If X denotes the number of white balls, then find the probability distribution of X.	10+10
Q.No.3	a) Construct the discrete probability distribution which corresponds to the experiment of the single roll of a pair of dice. Assume equal likelihood of occurrence of each side of a die and let the random variable X equal the sum of the dots which appear on the pair. b) Find the following limits: (i) $\lim_{x\to 3}\sqrt{x^3 - 27}$ (ii) $\lim_{x\to -2}\left(\frac{10+x^2}{5-8x+2x^2}\right)$ (iii) $\lim_{x\to 5}\left[\left(\frac{x+3}{x+6}\right)(x^2 - 12)\right]$	10+10
Q.No.4	The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (Stated in dollars) and the number of units produced x is $P = -0.01x^2 + 5{,}000x - 25{,}000$ (i) Determine the number of units x which will result in maximum profit. (ii) What is the expected maximum profit?	

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Q.No.5	Find $f'(x)$, $f''(x)$, $f'''(x)$, $f'(1)$, $f''(1)$, $f''(5)$ and $f'''(5)$ when (i) $f(x) = \frac{2x}{x^2+1}$ (ii) $f(x) = e^{2x}\ln x$	20
Q.No.6	Find A^{-1} and show that $AA^{-1} = I$, if $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{bmatrix}$	20
Q.No.7	a) Solve the following second degree inequality $x^2 + 4x - 12 \leq 0$ b) Solve graphically and check your answer algebraically. $-x + 3y = 2$ $4x - 12 = -8$	10+
Q.No.8	a) Determine the location and values of the absolute maximum and absolute minimum for the given function: $f(x) = (-6x + 7)^4$, where $0 \leq x \leq 3$ b) Sketch the graph of the function: $e^{-2x} - 4x$.	10

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Level	Bachelor/AD	Semester	Autumn 2021
Paper	Business Mathematics (1429)	Total Marks	100
Time Allowed	03 Hours	Pass Marks	50

NOTE: ATTEMPT ANY FIVE QUESTIONS.

Q.No	Questions	Marks
Q.No.1	a. Define and explain discrete and continuous random variables with the help of examples. b. Define and explain the following terms i. Consistent system of equations ii. Inconsistent system of equations	20
Q.No.2	a. Solve $\frac{x+1}{x-3} - 1 \leq 2$ b. Show that the equations $2x - y + z = 7$ $3x + y - 3z = 13$ $x + y + z = 5$ Are consistent and have unique solution	20
Q.No.3	a. If 2 fair dices are thrown. What is the probability of getting: i. A sum of 8 or more dots ii. A double six b. Find the slope of the line segment joining the points [illegible] and [illegible] and also draw it.	20
Q.No.4	Solve the following linear inequalities and also draw them. i. $x + 1 \geq 0$ ii. $\lvert x \rvert \leq 1$	20
Q.No.5	For a charity show the rate of an adult pass is Rs.20 more than the children pass. When a company sold 30 adult and 200 children pass, the amount collected is Rs.2250. Find the price of an adult pass.	20
Q.No.6	a. Solve the following system of equations using Crammer's Rule. $x + y + z = 4$ $x - 2y + 3z = 5$ $y - z = -1$ b. What is the inverse of a 2 × 2 matrix A. Can we find the inverse of any given matrix A?	20
Q.No.7	A ball is droped from the roof of a building which is 256 m high. The height h of the ball at any instant t is described by $h(t) = -16t^2 + 256$ Find i. Instantaneous velocity of ball at $t = 3$ seconds ii. The time taken by ball to reach the ground level.	20
Q.No.8	a. Draw the following curve $f(x) = x^2 - 3x + 2$ b. Determine the critical points and their nature for the function $f(x) = \frac{1}{4}x^4 - \frac{9}{2}x^2$	20

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Le..ve..	Bachelor	Semester	Autumn 2021
Pa..r	Business Mathematics (1429)	Maximum Marks	100
Tim.. Allows	03 Hours	Pass Marks	50

Note: ATTEMPT ANY FIVE QUESTIONS. ALL CARRY EQUAL MARKS

Question # 01. **(10+10=20)**

a) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected from each group. Find the probability that the three selected consist of 1 girl and 2 boys

b) Solve: $|7x - 12| = |4 - 3x|$

Question # 02. **(10+10=20)**

(a) Construct the discrete probability distribution which corresponds to the experiment of the single roll of a pair of dice. Assume equal likelihood of occurrence of each side of a die and let the random variable X equal the sum of the dots which appear on the pair.

(b) Find the following limits:

(i) $\lim_{x\to 3} \sqrt{x^3 - 27}$ (ii) $\lim_{x\to -2} \left(\frac{10+x^2}{5-8x+2x^2}\right)$ (iii) $\lim_{x\to 5} \left[\left(\frac{x+3}{x+6}\right)(x^2 - 12)\right]$

Question # 03. **(12+8=20)**

(a) Solve the given system of equations by using Cramer's rule.

$$3x_1 + 2x_2 + x_3 = 11$$
$$5x_1 \quad - 3x_3 = 19$$
$$4x_2 + 6x_3 = 0$$

(b) The technological matrix for a three-industry input-output model is

$$\begin{bmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0.8 & 0.12 \\ 1 & 0.4 & 0 \end{bmatrix}$$

If the non-industry demand for the output of these industries is $d_1 = \$5$ million, $d_2 = \$3$ million, *and* $d_3 = \$4$ million, determine the equilibrium output levels for the three industries.

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Question # 04 (20)

The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (Stated in dollars) and the number of units produced x is

$$P = -0.01x^2 + 5{,}000x - 25{,}000$$

(i) Determine the number of units x which will result in maximum profit.

(ii) What is the expected maximum profit?

Question # 05. (20)

Find $f'(x)$, $f''(x)$, $f'''(x)$, $f'(1)$, $f''(1)$, $f''(5)$ and $f'''(5)$ when

(i) $f(x) = \frac{2x}{x^2+1}$ (ii) $f(x) = e^{2x} \ln x$

Question # 06. (20)

For a given function $f(x)$:

(i) find those open intervals over which the function is concave up, concave down.

(ii) find all points of inflection.

$$f(x) = x^4 + x^2 + 10$$

Question # 07. (10+10=20)

(a) Determine all second-order partial derivatives for

$$f(x, y) = \ln 3xy^3$$

(b) Discuss the validity of the following statements. If the statement is always true, explain why. If not, give examples.

i) If A and B are 2×2 diagonal matrices, then $A + B$ is a 2×2 diagonal matrix.

ii) If A and B are 2×2 diagonal matrices, then $A + B = B + A$.

iii) If A and B are 2×2 upper triangular matrices, then AB is a 2×2 upper triangular matrix.

iv) If A and B are 2×2 upper triangular matrices, then $AB = BA$.

Question # 08. (10+10=20)

(a) Given the demand function

$$q_1 = f(p_1, p_2) = 36{,}000 - 0.1p_1^2 - 0.5p_2^2$$

(i) Determine the partial derivatives f_{p_1} and f_{p_2}.

(ii) If $p_1 = 20$ and $p_2 = 10$, evaluate f_{p_1} and f_{p_2} and interpret their meaning.

(b) Sketch the graph of the function: $-e^{-x} - 10x$.

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Chapter # 1+2

Among the three coins, one has heads on both the faces the second has tails on both the faces, and the third has a head on one face and a tail on the other. A coin is chosen at random and tossed. The result is a head. What is the probability that the opposite face is a tail?

Let X be a random variable giving the number of aces in a random draw of 4 cards from an ordinary deck 52 cards. Construct a table showing the probability distribution of X.

The sample space of an experiment consists of five simple events E_1, E_2, E_3, E_4 and E_5. These events are mutually exclusive. The probabilities of occurrence of these events are $P(E_1) = .20$, $P(E_2) = .15$, $P(E_3) = .25$, $P(E_4) = .30$, $P(E_5) = .10$ Several compound events can be defined for this experiment. They are $F = \{E_1, E_2, E_3\}$, $G = \{E_1, E_3, E_5\}$, $H = \{E_4, E_5\}$ Determine (i) $P(G')$ (ii) $P(F \cup G)$ iii) $P(G \cup H)$ (iv) $P(F \cap H)$ (v) $P(F \cap G)$

In a survey of 410 salespersons and 350 construction workers, it is found that 164 of the salespersons and 196 of the construction workers were overweight. If a person is selected at random from the group, what is the probability that:

i) This person is overweight?
ii) This person is a salesperson, given that the person is overweight?
iii) This person is overweight, given that the person is a salesperson?
iv)This person is a construction worker, given that the person is not overweight?
v) This person is not overweight, given that the person is a construction worker?

Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One Child is selected from each group. Find the probability that the three selected consist of 1 girl and 2 boys.

Construct the discrete probability distribution which corresponds to the experiment of the single roll of pair of dice. Assume equal likelihood of occurrence of each side of a die and let the random variable X equal the sum of the dots which appear on the pair.

If 2 fair dices are thrown. What is the probability of getting:

i) A sum of 8 or more dots
ii) A double six

Suppose that E and F are events where $P(E) = 0.4$, $P(F) = 0.3$ and $P(E \cup F)=0.6$.

Determine: a) $P(E\cap F)$ (b) $P(E/F)$ (c) $P(F / E)$

Find a formula for the probability distribution of the number of boys in families with three children assuming equal probabilities for boys and girls.

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Chapter # 3+4

Solve the following

$|7x-12| = |4-3x|$ $|x - 12| = |4 - x|$

Solve the inequality

$|y+1| \leq -9$ $|x| \leq 1$

$x^2 + 4x - 12 \leq 0$ $\frac{x+1}{x-3} - 1 \leq 2$

$x + 1 \geq 0$

Solve graphically and check your answer algebraically.

$$-x + 3y = 24$$
$$x - 12 = -8$$

Solve graphically and check your answer algebraically.

$$-x + 3y = 2$$
$$4x - 12 = -8$$

Solve the following simultaneous linear equations by graphical method:

$$2x + 3y = 3$$
$$6x + 4y = 5$$

Solve the following simultaneous linear equations by graphical method.

$$2x + y = 3$$
$$x + 2y = 0$$

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i) Find the slope of the line segment joining the points (0,0) and (2,2) and also draw it.
ii) Determine the slope-intercept form of the linear equation which passes through the point (7, 2) and is parallel to the line $y = 8$.
iii) Determine the equation of the straight line which is perpendicular to the line $3x - 2y = -28$ and which passes through the point (5, -9).

Sketch the plane representing $0x + y + 4z = 4$

Sketch the graph of each of the following linear functions.

i) Graph passes through the point (-3,-2) with slope of -1.
ii) Graph passes through the point (2, 4) with slope of 0.
iii) Graph passes through the point (5, 0) with undefined slope
iv) Graph passes through the point (2,–1) with slope of 3..

Find the following :

i) What is y-intercept and slope of the equation $5x + 2y = 10?$
ii) Determine the equation of a straight line whose y-intercept is -3 and slope 4.
iii) Find the slope intercept form of the equation that passes through (-8, -4) and perpendicular to the line 8x - 2y = 0.
iv) Determine the equation of a straight line which has a slope of -5 and a y-intercept of (0, 15).

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Given the equation $\frac{x+y}{3} = 24 - x$

i) Rewrite the equation in slope intercept form

ii) Identify the slope and y– intercept

iii) Interpret the meaning of the slope

Find the equation of the line:

i) With x-intercept -3 and y-intercept 4.

ii) Through (- 8, 5) and parallel to the line $2x - 7y + 4 = 0$

iii) Through (8, - 3) having slope 0.

iv) Through (- 4, - 6) and perpendicular to the line having slope - 1/2

v) Through (5, - 8) and perpendicular to the line joining the points A(- 15, - 8) and B(10, 7)

Find the equation of

i) The vertical line through (-5, 3).

ii) The line with x-intercept -3 and y-intercept 4.

iii) Line through (-8, 5) and parallel to the line $2x + 7y + 4 = 0$

Consider the liner equation 3x-6y=24:

i) What is the slope of the line represented by the given equation?

ii) What is the slope of any line parallel to the given line?

iii) What is the slope of any line perpendicular to the given line?

iv) How many different lines are perpendicular to this line?

v) Find the equation of the line which is perpendicular to the given line and which passes through the point (2, 5).

A company manufactures two products, A and B. Each unit of A requires 3 labor hours and each unit of B requires 5 labor hours. Daily manufacturing capacity is 1500 labor hours.

i) If x units of Product A and y units of product B are manufactured each and all labor hours are to used, determine the linear equation that requires the use of 150 labor hours per day.

ii) How many units of A can be made each day if 25 units of B are manufactured each day?

iii) How many units of A can be made each week if 12 units of B are manufactured each day? (Assume a day work week.)

For a charity show the rate of an adult pass is RS.20 more than the children pass. When a company sold 50 adult and 200 children pass, the amount collected is Rs.2250. Find the price of an adult pass.

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$C = \frac{5}{9}F - \frac{160}{9}$ is an equation relating temperature in Celsius units to temperature measured on the Fahrenheit scale. Let c = degrees Celsius and F = degrees Fahrenheit; assume the equation is graphed with C measured on the vertical axis

i) Identify the slope and y-intercept
ii) Interpret the meaning of the slope and y intercept for purposes of converting from Fahrenheit to Celsius temperature.
iii) Solve the equation for F and rework parts i and ii if F is plotted on vertical axis.

In a certain marketplace the demand and supply functions for a commodity are as follows:

$$D : p = 100 - 5q$$
$$S : p = 20 + 4q$$

i) What are the initial equilibrium price and quantity?
ii) Assume an imaginative advertising campaign shifts the demand function two places to the right. Sketch the initial demand and supply functions. Sketch the new demand function. Find the new equilibrium price and quantity.
iii) Assume that a tax of $1 per unit is levied on the seller. What will be the effect on the supply function? Depict the situation graphically. Determine the new equilibrium price and quantity.

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Chapter # 5

If $A = \begin{pmatrix} 12 & 3 \\ -5 & 7 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 11 \\ 7 & 9 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & -8 \\ 13 & 9 \end{pmatrix}$, Verify that:

(i) A(BC) = (AB)C (ii) B(A+C) = BA + BC

Perform the matrix operation $\begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 0 & -4 \\ 3 & 6 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 0 & 5 \\ 6 & 2 & -2 & 0 \end{bmatrix}$

Find a, b, c, d so that

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

The technological matrix for a three-industries input-output model is

$$\begin{pmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0.8 & 0.12 \\ 1 & 0.4 & 0 \end{pmatrix}$$

If the non-industry demand for the output of these industries is d1=$5 million d2=$3 million, and d3=$4 million, determine the equilibrium output levels for the three industries.

Discuss the validity of the following statements. If the statement is always True explain why. if not, give examples.

i) If A and B are 2 x 2 diagonal matrices then $A+B$ is a 2 x 2 diagonal matrix.
ii) If A and B are 2 x 2 diagonal matrices then $A+B = B+A$
iii) If A and B are 2 x 2 upper triangular matrices then AB is a 2 x 2 upper triangular matrix.
iv) If A and B are 2 x 2 upper triangular matrices, then $AB = BA$.

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Chapter # 6

A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day (assume that an over dosage of either drug is nameless). Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of Substance N contains 2 units of drug A and 4 units of drug B. How many grams of substance M and N can be mixed to meet the minimum daily requirement?

Solve the given system of equations by using Cramer's rule.

$$3x_1 + 2x_2 + x_3 = 11$$
$$5x_1 - 3x_3 = 19$$
$$4x_2 + 6x_3 = 0$$

Solve the given system of equations by using Cramer's rule.

$$2x - y + z = 5$$
$$4x + 2y + 3z = 8$$
$$3x - 4y - z = 3$$

Solve the following system of equations using Crammer's Rule.

$$x + y + z = 4$$
$$x - 2y + 3z = 5$$
$$y - z = -1$$

Solve the given system of equations by using Cramer's rule.

$$2x_1 + 5x_2 + x_3 = 13$$
$$6x_1 - 7x_3 = 10$$
$$9x_2 + 8x_3 = 4$$

Solve the following system of equations by using Cramer's rule.

$$x_1+2x_2-3x_3=24 \quad 2x_1-4x_2+x_3=12 \quad 3x_1-6x_2+9x_3=16$$

Solve the given system of equations by using Cramer's rule.

$$x_1+2x_2+4x_3=5 \quad 4x_1-4x_2+6x_3=6 \quad 5x_1+7x_2+3x_3=7$$

What is the inverse of a 2 x 2 matrix A. Can we find the inverse of any given matrix A?

Determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ -2 & 3 & -1 \\ 5 & 4 & 2 \end{bmatrix}$ by using the matrix of cofactor approach.

Determine the inverse of $\begin{pmatrix} 11 & 4 & 7 \\ -7 & -3 & 1 \\ 5 & -6 & 9 \end{pmatrix}$ by using the matrix of cofactor approach.

Find A^{-1} and show that $AA^{-1} = I$, if

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{pmatrix}$$

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Chapter # 7

Find the following limits:

$\lim_{x\to 3}\sqrt{x^2-5}$ ii) $\lim_{x\to 8}\dfrac{\sqrt{x} - \sqrt{8}}{x-8}$ iii) $\lim_{x\to -2}\dfrac{3x}{(x+2)^3}$

Find the following limits:

$\lim_{x\to 3}\sqrt{x^3-27}$ ii) $\lim_{x\to -2}\left(\dfrac{10+x^2}{5-8x+2x^2}\right)$ iii) $\lim_{x\to 5}\left[\left(\dfrac{x+3}{x+6}\right)(x^2-12)\right]$

$lim_{x\to 3}(\dfrac{2x^2+7x-15}{x-3})$

Use a table of x and f(x) values to fund the indicated lime for the given function

$$\lim_{x\to 0}\frac{x^2+3x}{x}$$

The total sales S (in thousands of CD's) for a compact disk are given by,

$$S(t)=\frac{90t^2}{t^2+50}$$

Where t is the number of months since the release of the CD.

i) Find *S'(t)*.
ii) Find S(10) and *S'*(10). Write a brief verbal interpretation of these results.
iii) Use the result from part (ii) to estimate the total sales of these results

---

Find *f'(x) if*

$f(x)=\sqrt[3]{\dfrac{(1-x^3)^5x^3}{(9-2x^2)}}$ $f(x)=\dfrac{e^{x^3+1}}{\ln(x+1)}$

If f(x) = e^x ln(x), find f'(1) and f'(1).

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A firm sells each unit of a product for $50. The total cost of producing x (thousand) units is described by the function

$C(x) = 10 + 25x^2 + x^3$

where C(x) is measured in thousands of dollars.

(i) Use the marginal approach to determine the profit- maximizing level of output.

(ii) What is the total revenue at this level of output? Total cost? Total profit?

A firm estimates that it can sell Q units of its product with an advertising expenditure of x thousand dollars. where

$$Q = Q(x) = -x^2 + 600x + 25$$

i) Over what level of advertising expenditure is the number of units of product sold increasing?

ii) Over what level of advertising expenditure is the number of units of product sold decreasing?

A ball throw upward from the roof of a building which is 750 feet high will be at a height of h feet after t seconds as described by the function.

$$h = f(t) = -16t^2 + 50t + 750$$

(i) What is the height of a ball after 5 seconds?
(ii) What is the velocity of the ball after 5 seconds
(iii) What is the acceleration of the ball at t = 0? At = 5?

The annual profit for a firm depends upon the number of units produced. Specially the function which describe the relationship between profit *P* (stated in dollars) and the number of units produced *x* is

$$P = -0.01x^2 + 5{,}000x - 25{,}000$$

i) Determine the number of units x which will result in maximum profit.
ii) What is the expected maximum profit?

A ball is dropped from the roof of a building which is 256 m high. The height h of the ball at any instant t is described by

$$h(t) = -16t^2 + 256$$

Find

i. Instantaneous velocity of ball at t = 3 seconds
ii. The time taken by ball to reach ground level.

Chapter # 9

Define Critical Point. Determine the interval over which f is i) increasing ii) decreasing iii) neither increasing nor decreasing iv) concave up v) concave down. If

$$f(x) = \frac{2x^3}{3} - 5x^2 - \frac{1}{x} - 3$$

For a given function $f(x)$:

$$f(x) = x^4 + x^2 + 10$$

i) find those open intervals over which the function is concave up, concave down.
ii) find all points of inflection .

Define critical point, point of inflection. Find all relative extrema, using First Derivative Test. Sketch the graph of the function.

$$f(x) = \frac{x^2}{x-1}$$

Determine the location and values of the absolute maximum and absolute minimum for the given function.

$$f(x) = (-6x + 7)^4, \text{ where } 0 \le x \le 3$$

Determine the interval over which f is (i) increasing (ii) decreasing (iii) neither increasing nor decreasing (iv) concave up (v) concave down. If

$$f(x) = \frac{x^3}{3} - x^2 - 5$$

Determine the location of all critical points and determine their nature for the function:

$$f(x) = -100e^{-0.25x} - 50x$$

Identify the location of any inflection points for the following functions.

$$f(x) = \frac{3x^4}{12} + \frac{5x^3}{6} + 3x^2 + x$$

Sketch the graph of the function $f(x) = ln(x^2 + 25)$

Determine the interval over which the function

$$f(x) = 5x^2 - 20x + 100$$

(i) Increasing (ii) decreasing (iii) neither increasing nor decreasing

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$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ -8 & 5 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\, A}{|A|}$$

$$|A| = 2\begin{vmatrix} 2 & 3 \\ 5 & -5 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ -8 & -5 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -8 & 5 \end{vmatrix}$$

$= 2(-10-15) - 3(-5+24) + 0$

$= 2(-25) - 3(19)$

$= -50 - 57$

$= -107$

$a_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 5 & -5 \end{vmatrix} = -10-15 = -25$

$a_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ -8 & -5 \end{vmatrix} = -(-5+24) = -19$

$a_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ -8 & 5 \end{vmatrix} = 5+16 = 21$

$a_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 0 \\ 5 & -5 \end{vmatrix} = -(-15-0) = 15$

$a_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 0 \\ -8 & -5 \end{vmatrix} = (-10-0) = -10$

$a_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 3 \\ -8 & 5 \end{vmatrix} = -(10+24) = -34$

-34

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$$a_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 0 \\ 2 & 3 \end{vmatrix} = (9-0) = 9$$

$$a_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix} = -(6-0) = -6$$

$$a_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4-3 = 1$$

$$\text{Adj } A = \begin{bmatrix} -25 & -19 & 21 \\ 15 & -10 & -34 \\ 9 & -6 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} -25 & 15 & 9 \\ -19 & -10 & -6 \\ 21 & -34 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -25 & 15 & 9 \\ -19 & -10 & -6 \\ 21 & -34 & 1 \end{bmatrix}}{-107}$$

$$= \begin{bmatrix} -25/-107 & 15/-107 & 9/-107 \\ -19/-107 & -10/-107 & -6/-107 \\ 21/-107 & -34/-107 & 1/-107 \end{bmatrix}$$

Q1.

$f(x) = (-6x+7)^4$

where $0 \le x \le 3$

(i) find critical point
(ii) Evalute the function at C.P and End Point of the interval.

i→

$f(x) = (-6x+7)^4$

$f'(x) = 4(-6x+7)^3(-6)$

$f'(x) = -24(-6x+7)^3$

Now, find C.P

$-24(-6x+7)^3 = 0$

Since, $(-24) \neq 0$, so ignore it

$-6x+7 = 0$

$-6x = -7 \Rightarrow x = 7/6$

So, $x = \frac{7}{6}$ is critical point and end point of the interval.

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ii→ At $x = \frac{7}{6}$

a) $f\left(\frac{7}{6}\right) = \left[-6\left(\frac{7}{6}\right) + 7\right]^4$

At $x = 0$

b) $f(0) = [-6(0) + 7]^4 = 7^4$

c) at $x = 3$

$f(3) = [-6(3) + 7]^4$

$= [-11]^4$

Now calculate the values of function.

a) $f\left(\frac{7}{6}\right) = \left[-6\left(\frac{7}{6}\right) + 7\right]^4$

$= 1$

b) $f(0) = 7^4 = 2401$

c) $f(3) = (-11)^4 = 14641$

The maximum value is $f(3) = 14641$ and minimum value is $f\left(\frac{7}{6}\right) = 1$

So, within interval $(0 < x < 3)$

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→ The absolute max value is 14641,
which Occurse at $x = 3$

→ The absolute min value is 1,
which occurs at $x = \frac{7}{6}$

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(a) Solve second degree inequalities

$x^2 + 4x - 12 \leq 0$

Sol:-

Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$, $b = 4$, $c = -12$

$$x = \frac{(-4 \pm \sqrt{4^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 48}}{2}$$

$$x = \frac{-4 \pm \sqrt{64}}{2} = \frac{-4 \pm 8}{2}$$

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$x = 2 \quad ; \quad x = -6$

Now Create Sign chart
with 3 region
$(x < -6)$, $(x > 2)$, $(-6 \leq x \leq 2)$

(i) $x < -6$
choose test point $x = -7$
and Substitute it into
inequality
$(-7)^2 + 4(-7) - 12 \leq 0$
$49 - 28 - 12 \leq 0$
$9 \leq 0$

(ii)
$x > 2$
choose $x = 3$
$3^2 + 4(3) - 12 \leq 0$
$9 + 12 - 12 \leq 0$
$9 \leq 0$

(iii) $-6 \leq x \leq 2$
so, $x = 0$
$0^2 + 4(0) - 12 \leq 0$

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$-12 \leq 0$

Final Analyze the results

(i) $x < -6$, the inequality is not Satisfy $(9 \leq 0)$, So, it is false.

(ii) $(x > 2)$, the inequality is not Satisfied $(9 \leq 0)$ So, it's false.

(iii) $(-6 \leq x \leq 2)$. the inequality is Satisfied $(-12 \leq 0)$ So, it is true

This means that the values of x that Satisfy the inequality are between -6 and 2 including -6 and 2.

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(b) $-x + 3y = 2 \quad \rightarrow 1$

$4x - 12 = -8 \quad \rightarrow 2$

Sol:-

Algebraic Solution :-

1 →

$-x + 3 = 2$

$-x = 2 - 3 \Rightarrow -x = -1$

multiply b.s by (-1)

$x = 1$

2 →

$4x - 12 = -8$

$4x = -8 + 12$

$4x = 4$

$x = 1$

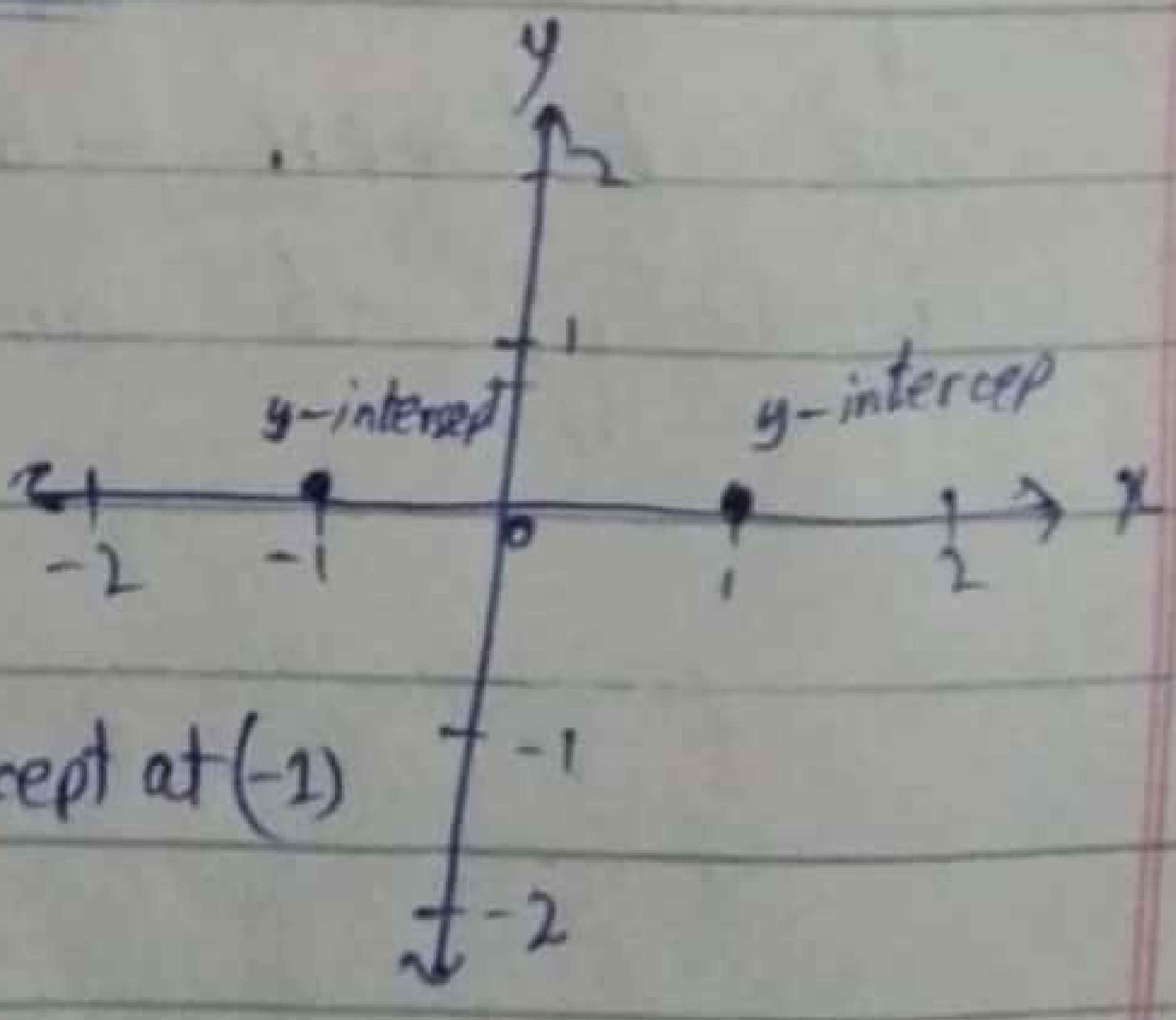
Graphic Solution :-

1 →

$-x + 3 = 2$

$-x = 2 - 3$

$-x = -1$



Slope of 1 and y-intercept at (-1)

2 → $4x - 12 = -8$

$4x = -8 + 12$

$4x = 4$

$x = 1$

Slope of 1 and y-intercept at (+1).

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$$f(x) = \frac{2x}{x^2+1}$$

Diff w.r.t "x"

$$f'(x) = \frac{x^2+1 \frac{d}{dx} 2x - 2x \frac{d}{dx} x^2+1}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(2) - (2x)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2 - 4x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{2 - 2x^2}{(x^2+1)^2}$$

$$f'(1) = \frac{2 - 2(1)^2}{((1)^2+1)^2} = \frac{2-2}{(2)^2}$$

$$= \frac{0}{4} = 0$$

$$f'(1) = 0$$

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$$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$$

Again diff w.r.t 'x'

$$f''(x) = \frac{(x^2+1)^2 \frac{d}{dx} 2-2x^2 - (2-2x^2)\frac{d}{dx}(x^2+1)^2}{((x^2+1)^2)^2}$$

$$= \frac{(x^2+1)^2(-4x) - (2-2x^2)\, 2(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)^2(-4x) - (2-2x^2)\, 4x\,(x^2+1)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)^2(-4x) - 2(1-x^2)\, 8x\,(x^2+1)}{(x^2+1)^4}$$

$$4x(x^2+1)\left[(x^2+1) - (1-x^2)2\right]$$
$$\overline{(x^2+1)^4}$$

$$4x\left[\frac{x^2+1-2+2x^2}{(x^2+1)^3}\right]$$

$$f''(x) = \frac{4x(3x^2-1)}{(x^2+1)^3}$$

$$f''(1) = \frac{4(1)(3(1)^2-1)}{((1)^2+1)^3}$$

$$= \frac{4(3-1)}{(1+1)^3} = \frac{4(2)}{8}$$

$$f''(1) = 1$$

$$f''(5) = \frac{4(5)(3(5)^2-1)}{((5)^2+1)^3}$$

$$\begin{array}{r} 25 \\ 3 \\ \hline 5 \end{array}$$

$$= \frac{20(75-1)}{(26)^3}$$

$$= \frac{20(74)}{26 \times 26 \times 26}$$

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$$f''(x) = \frac{12x^3 - 4x}{(x^2+1)^3}$$

Again diff w.r.t "x"

$$f'''(x) = \frac{(x^2+1)^3 \frac{d}{dx} 12x^3 - 4x - (12x^3 - 4x) \frac{d}{dx}(x^2+1)^3}{((x^2+1)^3)^2}$$

$$= \frac{(x^2+1)^3 (12(3x^2) - 4) - (12x^3 - 4x) 3(x^2+1)^2 (2x)}{(x^2+1)^6}$$

$$= \frac{(x^2+1)^3 (36x^2 - 4) - (12x^3 - 4x) 6x (x^2+1)^2}{(x^2+1)^6}$$

$$= \frac{(x^2+1)^2 [(x^2+1)(36x^2 - 4) - (12x^3 - 4x) 6x}{(x^2+1)^6}$$

$$= \frac{[(x^2+1)(36x^2 - 4) - (12x^3 - 4x) 6x}{(x^2+1)^4}$$

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SOLVED SELF ASSESMENT QUESTIONS

Q.1 Find $f_x, f_y, f_x(3,4)$ and $f_y(1,2)$.

(a) $f(x,y)=\dfrac{x+3xy^2}{x^3+y^2}$

Solution:

$$f(x,y)=\frac{x+3xy^2}{x^3+y^2}$$

$$f(x)=\frac{(x^3+y^2)\{1+3(1)y^2\}-(x+3xy^2)\{3x^2+0\}}{(x^3+y^2)^2}$$

$$=\frac{(x^3+y^2)(1+3y^2)-(x+3xy^2)\{3x^2\}}{(x^3+y^2)^2}$$

$$=\frac{x^3+3x^3y^2+y^2+3y^4-3x^3-3x^3y^2}{(x^3+y^2)^2}=\frac{-2x^3+y^2+3y^4}{(x^3+y^2)^2}$$

$$fx(3,4)=\frac{-2(3)^3+(4)^2+3(4)^4}{((3)^3+(4)^2)^2}=\frac{-2(27)+16+3(256)}{(27++16)^2}$$

$$=\frac{-54+16+768}{(43)^2}=\frac{730}{1849}=0.39$$

$$f(x,y)=\frac{x+3xy^2}{x^3+y^2}$$

$$fy=\frac{(x^3+y^2)\{0+3x(2y)\}-(x+3xy^2)\{0+2y\}}{(x^2+y^2)^2}$$

$$=\frac{(x^3+y^2)(6xy)-(x+3xy^2)(2y)}{(x^3+y^2)^2}=\frac{6x^4y+6xy^3-2xy-6xy^3}{(x^3+y^2)^2}$$

$$=\frac{(x^3+y^2)(6xy)-(x+3xy^2)(2y)}{(x^3+y^2)^2}=\frac{6x^4y+6xy^3-2xy-6xy^3}{(x^3+y^2)^2}$$

$$fy(1,2)=\frac{6x^4y-2xy}{(x^3+y^2)^2}$$

$$=\frac{6(1)^4(2)-2(1)(2)}{((1)^3+(2)^2)^2}=\frac{6(1)(2)-4}{(1+4)^2}=\frac{12-4}{(5)^2}=\frac{8}{25}=0.32$$

(b) $f(x, y) = ye^{(5x+2y^2)}$

Solution:

$$f(x,y) = y\ e^{(5x+2y^2)}$$

$$fx = y\ e^{(5x+2y^2)}.\{5(1)+0\} = y\ e^{(5x+2y^2)}(5) = 5y\ e^{(5x+2y^2)}$$

$$fx(3,4) = 5(4)\ e^{[5(x)+2(4)^2]} = 20\ e^{(15+32)} = 20\ e^{47}$$

$$f(x,y) = y.e^{(5x+2y^2)}$$

$$fy = (1).\ e^{(5x+2y^2)} + y.e^{(5x+2y^2)}\{0+2(2y)\}$$

$$= e^{(5x+2y^2)} + y.e^{(5x+2y^2)}(4y) = e^{(5x+2y^2)} + 4y.e^{(5x+2y^2)} = e^{(5x+2y^2)}[1+4y]$$

$$fy(1,2) = e^{(5(1)+2(2)^2)}(1+4(2)) = e^{(5+8)}(1+8) = 9\ e^{13}$$

(c) $f(x, y) = (x+1)^2 + (y-3)^3 + 5xy^3 - 2$

Solution:

$$f(x,y) = (x+1)^2 + (y-3)^3 + 5xy^3 - 2$$

$$fx = 2(x+1)^{2-1}(1+0) + 0 + 5(1)y^3 - 0 = 2(x+1) + 5y^3$$

$$fx(3,4) = 2(3+1) + 5(4)^3$$

$$= 2(4) + 5(64) = 8 + 320 = 328$$

$$f(x,y) = (x+1)^2 + (y-3)^3 + 5xy^3 - 2$$

$$fy = 0 + 3(y-3)^{3-1}(1-0) + 5x(3y^2) - 0 = 3(y-3)^2 + 15xy^2$$

$$fy(1,2) = 3(2-3)^2 + 15(1)(2)^2$$

$$= 3(-1)^2 + 15(1)(4) = 3 + 60 = 63$$

(d) $f(x,y) = \dfrac{\sqrt{x^3+4}}{x^2y+y^2x}$

Solution:

$$f(x,y) = \frac{\sqrt{x^3+4}}{x^2y+y^2x}$$

$$fx = \frac{(x^2y+y^2x)\left\{\frac{1}{2}(x^2+4)^{-\frac{1}{2}}(2x)\right\} - \sqrt{x^2+4}\{(2x)y+y^2(1)}{(x^2y+y^2x)^2}$$

$$= \frac{(x^2y+y^2x)\left\{\frac{x}{\sqrt{x^2+4}}\right\} - \sqrt{x^2+4}(2xy+y^2)}{(x^2y+y^2x)^2}$$

$$= \frac{\dfrac{x\left(x^2y+y^2x\right)-\left(x^2+4\right)\left(2xy+y^2\right)}{\sqrt{x^2+4}}}{\left(x^2y+y^2x\right)^2}$$

$$= \frac{x^3y+x^2y^2-2x^3y-x^2y^2-8xy-4y^2}{\sqrt{x^2+4}\left(x^2y+y^2x\right)^2} = \frac{-x^3y-8xy-4y^2}{\sqrt{x^2+4}\left(x^2y+y^2x\right)^2}$$

$$fx\,(3,4) = \left[\frac{-(3)^3(4)-8(3)(4)-4(4)^2}{\sqrt{(3)^2+4}\left[\left((3)^2(4)+(4)^2(3)\right)\right]^2}\right]$$

$$= \frac{-(27)(4)-96-4(16)}{\sqrt{9+4}\left[(9)(4)+(16)(3)^2\right]} = \frac{-108-96-64}{\sqrt{13}\,[36+48]^2}$$

$$= \frac{-268}{(3.61)(84)^2} = \frac{-268}{25472.16} = -0.011$$

$$f(x,y) = \frac{\sqrt{x^2+4}}{x^2y+y^2x}$$

$$fy = \frac{\left(x^2y+y^2x\right)\{0\}-\sqrt{x^2+4}\left\{x^2(1)+(2y)x\right\}}{\left(x^2y+y^2x\right)^2}$$

$$= \frac{0-\sqrt{x^2+4}\left(x^2+2xy\right)}{\left(x^2y+y^2x\right)^2} = \frac{\sqrt{-x^2+4}\left(x^2+2xy\right)}{\left(x^2y+y^2x\right)^2}$$

$$fy\,(1,2) = \frac{-\sqrt{(1)^2+4}\left[(1)^2+2(1)(2)\right]}{\left[(1)^2(2)+(2)^2(1)\right]^2} = \frac{-\sqrt{1+4}\,[1+4]}{\left[(1)^2(2)+(2)^2(1)\right]^2} = \frac{-\sqrt{5}\,(5)}{(2+4)^2}$$

$$= \frac{-(2.24)(5)}{(6)^2} = \frac{-11.2}{36} = -0.31$$

(e) $f(x,y) = x^4\,\ell n(1+xy^{\frac{3}{2}})$

Solution :

$$f(x,y) = x^4\,\ell n\left(1+xy^{\frac{3}{2}}\right)$$

$$fx = x^4\left\{\frac{1}{1+xy^{\frac{3}{2}}}\cdot\left(0+(1)y^{\frac{3}{2}}\right)\right\}+\ell n\left(1+xy^{\frac{3}{2}}\right)\left\{4x^3\right\}$$

$$= \frac{x^4y^{\frac{3}{2}}}{\left(1+xy^{\frac{3}{2}}\right)}+4x^3\ell n\left(\left[1+xy^{\frac{3}{2}}\right]\right)$$

$$fx(3,4)=\frac{(3)^4(4)^{\frac{3}{2}}}{\left[1+(3)(4)^{\frac{3}{2}}\right]}+4(3)^3\,\ell n\left[1+(3)(4)^{\frac{3}{2}}\right]$$

$$=\frac{(81)(8)}{(1+24)}+4(27)\,\ell n[1+24]$$

$$=\frac{648}{25}+108\,\ell n(25)=25.92+347.64=373.56$$

$$f(x,y)=x^4\,\ell n\left(1+xy^{\frac{3}{2}}\right)$$

$$fy=x^4.\left\{\frac{1}{1+xy^{\frac{3}{2}}}\left(0+x.\left(\frac{3}{2}x^{\frac{1}{2}}\right)\right)\right\}=\frac{x^4\left(\frac{3}{2}x^{\frac{3}{2}}\right)}{\left(1+xy^{\frac{3}{2}}\right)}=\frac{3/2\,x^{\frac{11}{2}}}{\left(1+xy^{\frac{3}{2}}\right)}$$

$$fy(1,2)=\frac{3/2\,(1)^{\frac{11}{2}}}{1+(1)(2)^{\frac{3}{2}}}=\frac{3/2}{1+2.83}=\frac{1.5}{3.83}=0.39$$

(f) $f(x,y)=-\dfrac{2}{e^{(4x-xy^2+y)}}$

Solution:

$$f(x,y)=\frac{-2}{e^{(4x-xy^2+y)}}=-2e^{-(4x-xy^2+y)}$$

$$fx=-2e^{-(4x-xy^2+y)}\{-4(1)-(1)y^2+0\}=-2e^{-(4x-xy^2+y)}\{-4-y^2\}$$

$$fx(3,4)=-2e^{-\left[4(3)-(3)(4)^2+4\right]}\left(-4-(4)^2\right)$$

$$=-2e^{-(12-48+4)}(-4-16)=-2e^{-(-32)}\{-20\}=40e^{32}$$

$$f(x,y)=-\frac{2}{e^{(4x-xy^2+y)}}$$

$$fy=-2e^{-(4x-xy^2+y)}\{-(0-x(2y)+1)\}=-2e^{-(4x-xy^2+y)}(2xy-1)$$

$$fy(1,2)=-2e^{-(4(1)-(1)(2)^2+1)}\{2(1)(2)-1\}$$

$$=-2e^{-(4-4+1)}(4-1)=-2e^{-1}(3)=-\frac{6}{e}$$

☆☆☆☆☆

Q2: If $z = x^2 - y^2 + 2xy$ then show that.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Solution :

$$z = x^2 - y^2 + 2xy$$

$$\frac{\partial^2 z}{\partial x^2} = 2x - 0 + 2(1) = 2x + 2y$$

$$\frac{\partial^2 z}{\partial x^2} = 2(1) + 0 = 2$$

$$Z = x^2 - y^2 + 2xy$$

$$\frac{\partial z}{\partial y} = 0 - 2y + 2x(1) = -2y + 2x$$

$$\frac{\partial^2 z}{\partial x^2} = -2(1) + 0 = -2$$

$$L.H.S = \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} = 2 + (-2) = 2 - 2 = 0 = R.H.S$$

Hence, $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

☆☆☆☆☆

Q3: Let $f(x, y, z) = x^3 y^5 z^7 + xy^2 + y^3 z - x^2 z^2$. **Find**

Solution :

$$f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z - x^2z^2$$

$$f(x) = (3x^2)y^5z^7 + (1)y^2 + 0 - (2x)z^2$$

$$= 3x^2y^5z^7 + y^2 - 2xz^2$$

(a) fxz

$$fxz = 3x^2y^5(7z^6) + 0 - 2x(2z)$$

$$= 21x^2y^5z^6 - 4xz$$

$$fy = x^3(5y^4)z^7 + x(2y) + (3y^2)z - 0$$

$$= 5x^3y^4z^7 + 2xy + 3y^2z$$

(b) fyz

Solution :

$$fyz = 5x^3y^4(7z^6) + 0 + 3y^2(1)$$

$$= 35x^3y^4z^6 + 3y^2$$

(d) fxy

$$fxz = 3x^2(5y^4)z^6 + 2y - 0 = 15x^2y^4z^6 + 2y$$

(e) fxyz

$$fxyz = 15x^2y^4(6z^5) + 0 = 90x^2y^4z^5$$

(e) fyzx

$$fyzx = 35(3x^2)y^4z^6 + 0 = 105x^2y^4z^6$$

(f) fxzy

$$fxzy = 21x^2(5y^4)z^6 - 0 = 105x^2y^4z^6$$

(g) fxxy

$$fx = 3x^2y^5z^7 + y^2 - 2xz^2$$

$$fxx = 3(2x)y^5z^7 + 0 - 2(1)z^2 = 6xy^5z^7 - 2z^2$$

$$fxxy = 6x(5y^4)z^7 - 0 = 30xy^4z^7$$

(h) fxxz

$$fxxz = 6xy^5(7z^6) - 2(2z) = 42xy^5z^6 - 4z$$

(i) fyyx

$$fy = 5x^3y^4z^7 + 2xy - 3y^2z$$

$$fyy = 5x^3(4y^3)z^7 + 2x(1) - 3(2y)z = 20x^3y^3z^7 + 2x - 6yz$$

$$fyyz = 20x^3y^3(7z^6) + 0 - 6y(1) = 140x^3y^3z^6 - 6y$$

(j) fyzy

$$fyzy = 35x^3(4y^3)z^6 + 3(2y) = 140x^3y^3z^6 + 6y$$

☆☆☆☆☆

Q4: Suppose z = f(x, y) represents the cost to construct a certain structure, where x and y express the labor and material cost respectively. Explain what f_x and f_y represent.

Solution :

Here z = f (x, y)

And zx = fx represents to construct a certain structure when labour may be variable (increased or decreased).

And zy = fy represents the cost to construct a certain structure when material cost may be variable (increased or decreased).

☆☆☆☆☆

$$fxy(x,y) = (2y - 0)e^x = 2ye^x$$

$$fxy(0,0) = 2(0)e^0 = 2(0)(1) = 0$$

And the relative minimum value is given by

$$D = fxx(x,y) \,.\, fyy(x,y) - fxy^2(x,y)$$

$$= fxx(0,0) \,.\, fyy(0,0) - fxy^2(0,0)$$

$$= (-4)(0) - (0)^2 = 0 - 0 = 0$$

Since D = 0, then this test fails and no conclusion is to be taken

(b) $f(x, y) = x^2 + y^2 + xy - 9x + 1$

Solution :

$$f(x,y) = x^2 + y^2 + xy - 9x + 1$$

$$fx(x,y) = 2x + 0 + (1)y - 9(1) + 0 = 2x + y - 9$$

Put $fx(x, y) = 0$

$$2x + y - 9 = 0$$

$$2x + y = 9 \quad _____ (i)$$

$$fy(x,y) = 0 + 2y + x(1) - 0 + 0 = 2y + x$$

Put $fy(x, y) = 0$

$$2y + x = 0$$

$$x + 2y = 0$$

$$x = -2y \quad _____ (ii)$$

Put eq. (ii) in eq. (i), we get

$$-2y + y = 9$$

$$-y = 9$$

$$y = -9$$

Put y = -9 in eq. (ii), we get

$$x = -2(9)$$

$$x = -18$$

Hence, (-18, -9) is the only critical point

Now

$$fxx(x,y) = 2(1) + 0 - 0 = 2$$

$$fxx(-18,-9) = 2 > 0 \qquad \text{(Minimum)}$$

$$fyy(x,y) = 2(1) + 0 = 2$$

$$fyy(-18,-9) = 2$$

$$fxy(x,y) = 2(0) + 1 - 0 = 1$$

Q5: The joint cost function for producing x units of product A and y units of product B is given by.

$$e=\frac{x^2(y^3+x)^{\frac{1}{2}}}{17}+xy^{\frac{1}{2}}+600$$

Find the marginal costs $\frac{\partial c}{\partial x}$ and $\frac{\partial c}{\partial y}$ when x = 17 and y = 8 and interpret the result.

Solution :

$$e=\frac{x^2(y^3+x)^{\frac{1}{2}}}{17}+xy^{\frac{1}{2}}+600$$

$$\frac{\partial e}{\partial x}=\frac{1}{17}\left[x^2\left\{\frac{1}{2}(y^3+x)^{-\frac{1}{2}}.(0+1)\right\}+\left(y^3+x^{\frac{1}{2}}\right)\{2x\}+(1)y^{\frac{1}{2}}+0\right]$$

$$=\frac{1}{17}\left[\frac{x^2}{2(y^3+x)^{\frac{1}{2}}}+2x(y^3+x)^{\frac{1}{2}}\right]+y^{\frac{1}{2}}$$

Put x = 17, y = 8 in above equation

$$=\frac{1}{17}\left[\frac{(17)^2}{2((8)^3+17)^{\frac{1}{2}}}+2(17)\left[(8)^3+17\right]^{\frac{1}{2}}\right]+(8)^{\frac{1}{2}}$$

$$=\frac{1}{17}\left[\frac{289}{2[512+17]^{\frac{1}{2}}}+34(512+17)^{\frac{1}{2}}\right]+2.83=\frac{1}{17}\left[\frac{289}{2(529)^{\frac{1}{2}}}+34(529)^{\frac{1}{2}}\right]+2.83$$

$$=\frac{1}{17}\left[\frac{289}{2(23)}+34(23)\right]+2.83=\frac{1}{17}\left[\frac{289}{46}+782\right]+2.83$$

$$=\frac{1}{17}[6.28+782]+2.83=\frac{1}{17}[788.28]+2.83=46.37+2.83=49.20$$

Now

$$e=\frac{x^2(y^3+x)^{\frac{1}{2}}}{17}+xy^{\frac{1}{2}}+600$$

$$\frac{\partial e}{\partial y}=x^2\left\{\frac{1}{2}(y^3+x)^{-\frac{1}{2}}(3y^2+0)\right\}+x\left\{\frac{1}{2}y^{-\frac{1}{2}}\right\}+0$$

$$=\frac{3x^2y^2}{2(y^3+x)^{\frac{1}{2}}}+\frac{x}{2y^{\frac{1}{2}}}$$

Put x = 17 and y = 8 in above equation

$$= \frac{3(17)^2(8)^2}{2\left[(8)^3+17\right]^{\frac{1}{2}}} + \frac{17}{2(8)^{\frac{1}{2}}} = \frac{3(289)(64)}{2[512+17]^{\frac{1}{2}}} + \frac{17}{2(2.83)}$$

$$= \frac{55488}{2(529)^{\frac{1}{2}} + \frac{17}{5.66}} = \frac{55488}{2(23)} + 3 = \frac{55488}{46} + 3 = 1206.26 + 3 = 1209.26$$

☆☆☆☆☆

Q6: Find all points where functions have any relative extrema.

(a) $f(x, y) = (y^2 - 4)(e^x - 1)$

Solution :

$$f(x, y) = (y^2 - 4)(e^x - 1)$$

$$fx(x, y) = (y^2 - 4)(e^x(1) - 0) = (y^2 - 4)(e^x)$$

Put $fx(x, y) = 0$

$$(y^2 - 4)(e^x) = 0$$

$\Rightarrow$ $(y^2 - 4) = 0$ or $(e^x) = 0$ (Not possible)

$$y^2 = 4$$

$$y = \pm 2$$

$$fy(x, y) = (2y - 0)(e^x - 1)$$

$$= 2y(e^x - 1)$$

Put $fy(x, y) = 0$

$$2y(e^x - 1) = 0$$

$\Rightarrow$ $2y = 0$ or $e^x - 1 = 0$

$y = 0$ $\quad e^x = 1$

$$e^x = e^0$$

$$\Rightarrow x = 0$$

Hence, (0, 0) is the only critical point

Now $fxx(x, y) = (y^2 - 4)(e^x)$

$$fxx(0, 0) = (0 - 4)(e^0)$$

$$= (-4)(1) = -4 < 0 \quad \text{(Minimum)}$$

$$fyy(x, y) = 2(1)(e^x - 1) = 2(e^x - 1)$$

$$f(yy(0, 0)) = 2(e^0 - 1) = 2(1 - 1) = 2(0) = 0$$

$$fxy(x,y) = (2y-0)e^x = 2ye^x$$

$$fxy(0,0) = 2(0)e^0 = 2(0)(1) = 0$$

And the relative minimum value is given by

$$D = fxx(x,y)\,.\,fyy(x,y) - fxy^2(x,y)$$

$$= fxx(0,0).\,fyy(0,0) - fxy^2(0,0)$$

$$= (-4)(0) - (0)^2 = 0 - 0 = 0$$

Since D = 0, then this test fails and no conclusion is to be taken

(b) $f(x, y) = x^2 + y^2 + xy - 9x + 1$

Solution :

$$f(x,y) = x^2 + y^2 + xy - 9x + 1$$

$$fx(x,y) = 2x + 0 + (1)y - 9(1) + 0 = 2x + y - 9$$

Put $fx(x, y) = 0$

$$2x + y - 9 = 0$$

$$2x + y = 9 \quad ____ (i)$$

$$fy(x,y) = 0 + 2y + x(1) - 0 + 0 = 2y + x$$

Put $fy(x, y) = 0$

$$2y + x = 0$$

$$x + 2y = 0$$

$$x = -2y \quad ____ (ii)$$

Put eq. (ii) in eq. (i), we get

$$-2y + y = 9$$

$$-y = 9$$

$$y = -9$$

Put y = -9 in eq. (ii), we get

$$x = -2(9)$$

$$x = -18$$

Hence, (-18, -9) is the only critical point

Now

$$fxx(x,y) = 2(1) + 0 - 0 = 2$$

$$fxx(-18,-9) = 2 > 0 \quad \text{(Minimum)}$$

$$fyy(x,y) = 2(1) + 0 = 2$$

$$fyy(-18,-9) = 2$$

$$fxy(x,y) = 2(0) + 1 - 0 = 1$$

And the relative minimum value is given by

$$D = fxx(x,y).fyy(x,y) - fx^2y(x,y)$$
$$= fxx(-18,-9).fyy(-18,-9) - fx^2y(-18,-9)$$
$$= (2)(2) - (1)^2 = 4 - 1 = 3 > 0$$

Since $D > 0$ and $fxx(x, y) > 0$, then the function has a relative minimum at the point $(-18,-9)$.

(i) $f(x, y) = 4xy - x^4 y^4$

Solution :

$$f(x,y) = 4xy - x^4 - y^4$$
$$fx(x,y) = 4(1)y - 4x^3 - 0 = 4y - 4x^3$$

Put $f(x,y) = 0$

$$4y - 4x^3 = 0$$
$$4(y - x^3) = 0$$
$$\Rightarrow \quad y - x^3 = 0 \quad \text{_____(i)}$$
$$fy(x,y) = 4x(1) - 0 - 4y^3 = 4x - 4y^3$$

Put $fy(x,y) = 0$

$$4x - 4y^3 = 0$$
$$4(x - y^3) = 0$$
$$\Rightarrow \quad x - y^3 = 0 \quad \text{_____(ii)}$$

Equating eq. (i) and eq. (ii), we get

$$x - y^3 = y - x^3$$
$$x - y = y^3 - x^3$$
$$(x - y) = -(x^3 - y^3)$$
$$(x - y) = -(x - y)(x^2 + xy + y^2)$$
$$1 = -(x^2 + xy + y^2)$$
$$x^2 + xy + y^2 + 1 = 0$$
$$x^2 + xy + xy + y^2 + 1 - xy = 0$$
$$x^2 + 2xy + y^2 = xy - 1$$
$$(x + y)^2 = xy - 1$$
$$x + y = \pm\sqrt{xy + 1}$$
$$x = -y \pm \sqrt{xy + 1}$$

The above equation does not generate real solution. Hence, there is no solution

(d) $f(x,y) = x^2 + y^2 + \frac{32}{xy}$

Solution :

$$f(x,y) = x^2 + y^2 = \frac{32}{xy}$$

$$fx(x,y) = 2x + 0 + \frac{32}{xy}((-1)x^{-1-1})$$

$$= 2x + \frac{32}{y}(-x^{-2}) = 2x - \frac{32}{x^2y}$$

Put $fx(x,y) = 0$

$$2x - \frac{32}{x^2y} = 0$$

$$\frac{2x^3y - 32}{x^2y} = 0$$

$$\Rightarrow \quad 2x^3v - 32 = 0$$

$$2(x^3y - 16) = 0$$

$$\Rightarrow \quad x^3y - 16 = 0 \quad \text{_____(i)}$$

$$fy(x,y) = 0 + 2y + \frac{32}{x}((-1)y^{-1-1})$$

$$= 2y + \frac{32}{x}(-y^{-2}) = 2y - \frac{32}{xy^2}$$

Put $fy(x,y) = 0$

$$2y - \frac{32}{xy^2} = 0$$

$$\frac{2xy^3 - 32}{xy^2} = 0$$

$$\Rightarrow \quad 2xy^3 - 32 = 0$$

$$2(xy^3 - 16) = 0$$

$$\Rightarrow \quad xy^3 - 16 = 0 \quad \text{_____(ii)}$$

Equating eq. (i) and (ii), we get

$$xy^3 - 16 = xy^3 - 16$$

$$x^3y = xy^3$$

$$x^2 = y^2$$

$$\sqrt{x^2} = \pm\sqrt{y^2}$$

$$x = \pm y$$

Put $x = y$ in eq. (i), we get

$$y^3.y-16=0$$
$$y^4=16$$
$$y^4=2^4$$
$$\Rightarrow \quad y=2$$

Put y = 2 in x = y, we get

$$x=2$$

Put $x=-y$ in eq. (i), we get

$$(-y)^3.y-16=0$$
$$-y^4-16=0$$
$$y^4=16$$
$$y^4=-2^4$$

Put $y=-2$ in $x=-y$, we get

$$x=-(-2)$$
$$x=2$$

Hence (2, 2) and (2, -2) are the critical points

$$fxx(x,y)=2(1)-\frac{32}{y}(-2x^{-3})=2+\frac{64}{x^3y}$$
$$fxx(2,2)=2+\frac{64}{(2)^3(2)}$$
$$=2+\frac{64}{16}=2+4=6>0 \quad \text{(Minimum)}$$
$$fyy(x,y)=2(1)-\frac{32}{x}(-2y^{-3})=2+\frac{64}{xy^3}$$
$$fyy(2,2)=2+\frac{64}{(2)(2)^3}=2+\frac{64}{16}=2+4=6$$
$$fxy(x,y)=2+(0)-\frac{32}{x^2}(-y^{-2})=0+\frac{32}{(2)^2(2)^2}$$
$$fxy(2,2)=\frac{32}{(2)^2(2)^2}=\frac{32}{16}=2$$

And the relative minimum value is given by

$$D=fxx(x,y).fyy(x,y)-fx^2y(x,y)$$
$$=fxx(2,2)fyy(2,2)-fx^2y(2,2)$$
$$=(6)(6)-(2)^2=36-4=32>0$$

Since D > 0 and fxx(2, 2) > 0, then the function has a relative minimum at the point (2, 2).

$$fxx(2,-2) = 2 + \frac{64}{(2)^3(-2)} = 2 + \frac{64}{(8)(-2)}$$

$$= 2 - \frac{64}{16} = 2 - 4 = -2 < 0 \qquad \text{(Maximum)}$$

$$fyy(2,-2) = 2 + \frac{64}{(2)^3(-2)^3} = 2 + \frac{64}{(2)(-8)}$$

$$= 2 - \frac{64}{16} = 2 - 4 = -2$$

$$fxy(2,-2) = \frac{32}{(2)^2(-2)^2} = \frac{32}{16} = 2$$

And the relative maximum value is given by

$$D = fxx(x,y).\ fyy(x,y) - fx^2y(x,y)$$

$$= fxx(2,-2).\ fyy(2,-2) - fx^2y(2,-2)$$

$$= (-2)(-2) - 2 = 4 - 2 = 2 > 0$$

Since $D > 0$ and $fxx(2,-2) < 0$, then the function has a relative maximum at the point $(2,-2)$.

(e) $f(x, y) = xe^{(y+1)}$

Solution :

$$f(x,y) = x\ e^{(y+2)}$$

$$fx(x,y) = (1)e^{(y+2)} = e^{(y+2)}$$

Put $f(x,y) = 0$

$$e^{(y+2)} = 0 \qquad \text{(Not possible)}$$

$$fx(x,y) = x.e^{(y+2)}(1+0) = x\ e^{(y+2)}$$

Put $fy(x,y) = 0$

$$x\ e^{(y+2)} = 0$$

$\Rightarrow$ $x = 0$ or $e^{(y+2)} = 0$ (Not possible)

Hence, the solution is not possible.

☆☆☆☆☆

Q7: Suppose a production function P is given by.

$$P = f(l,k) = 0.8l^3 - 0.02l^2 + 2.5k^2 - 0.05k^3$$

Find the values of l and k so as to maximize output P.

Solution :

$$P = f(t,k) = 0.8t^3 - 0.02t^2 + 2.5k^2 - 0.05k^3$$

$$f_t(t,k) = 0.8t^2 - 0.0(2t) + 0 - 0 = 2.4t^2 - 0.04t$$

Put $f_t(t,k)=0$

$$2.4t^2-0.04t=0$$
$$t(2.4t-0.04)=0$$
$$\Rightarrow \quad t=0 \quad \text{or} \quad 2.4t-0.04=0$$
$$2.4t=0.04$$
$$t=\frac{0.04}{2.4}=\frac{1}{60}=0.02$$
$$f_k(t,k)=0-0+0.02(2k)-0.05(3k^2)$$
$$=0.04k-0.15k^2$$

Put $f_k(t,k)=0$

$$0.04k-0.15k^2=0$$
$$k(0.04-0.15k)=0$$
$$\Rightarrow \quad k=0 \quad \text{or} \quad 0.04k-0.15k=0$$
$$0.15k=0.04$$
$$k=\frac{0.04}{0.15}=0.27$$

Hence, (0, 0) and (0.02, 0.27) are the critical points

$$f_{tt}(t,k)2.4(2t)-0.04(1)=4.8t-0.04$$
$$f_{tt}(0,0)=4.8(0)-0.04(1)=-0.04<0 \qquad \text{(Maximum)}$$
$$f_{kk}(t,k)=0.04(1)-0.15(2k)=0.04-0.30k$$
$$f_{kk}(0,0)=0.04-0.(30)(0)=0.04$$
$$f_{tk}(t,k)=0$$

And the relative maximum value at (0, 0) is given by

$$D=f_{tt}(0,0)f_{kk}(0,0)-f_{t^2k}(0,0)$$
$$=(-0.04)(0.04)-(0)^2$$
$$=-0.0016$$

☆☆☆☆☆

Q8: Suppose that the profit of a certain company is given by.

$$P(x,y)=1200+80x-2x^2+100y-y^2$$

Where x and y represent the cost of unit of labor and unit of goods respectively. Find x and y that maximize the profit. Find the maximum profit.

Solution :

$$P(x,y)=1200+80x-2x^2+100y-y^2$$
$$Px(x,y)=0+80(1)-2(2x)-0-0=80-4x$$

Put $Px(x, y) = 0$

$80 - 4x = 0$

$4x = 80 \Rightarrow x = \frac{80}{4} = 20$

$Py(x, y) = 0 + 0 - 0 + 100(1) - 2y = 100 - 2y$

Put $Py(x, y) = 0$

$100 - 2y = 0$

$2y = 100 \Rightarrow x = \frac{100}{2} = 50$

Hence, (20, 50) is the only critical point

$Pxx(x, y) = 0 - 4(1) = -4 < 0$ (Maximum)

$Pyy(x, y) = 0 - 2(1) = -2$

$Pxy(x, y) = 0 - 0 = 0$

And the relative maximum at (20, 50) is

$$D = Pxx(20,50) . Pyy(20,50) - Px^2y(20,50) = (-4)(-2) - (0)^2 = 8$$

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Q9: The revenue in thousands of dollars obtained from selling the x spas and y solar heaters is given by.

$$R(x, y) = 15 + 80x + 81y - 2x^2 - 3y^2 - 4xy$$

Find the quantity of each that must be sold to generate maximum revenue. Find the maximum revenue.

Solution:

$R(x, y) = 15 + 80x + 81y - 2x^2 - 3y^2 - 4xy$

$Rx(x, y) = 0 + 80(1) + 0 - 2(2x) - 0 - 4(1)y = 80 - 4x - 4y$

Put $Rx(x, y) = 0$

$80 - 4x - 4y = 0$

$4(20 - x - y) = 0$

$\Rightarrow$ $20 - x - y = 0$

or $x + y = 20$ ____(i)

$Ry(x, y) = 0 + 0 + 81(1) - 0 - 3(2y) - 4x(1) = 81 - 6y - 4x$

Put $Ry(x, y) = 0$

$81 - 6y - 4x = 0$

$4x + 6y = 81$ ______(ii)

$4(i) - (ii)$, gives

$$\begin{array}{r} 4x+4y=80 \\ \pm 4x \pm 6y = \pm 81 \\ \hline -2y=-1 \end{array}$$

$$y=\frac{1}{2}=0.5$$

Put y = 0.5 in eq. (i), we get

$$x+0.5=20$$

$$x=20-0.5=19.5$$

Hence (19.5, 0.5) is the only critical point

$$Rxx(x,y)=0-4(1)-0=-4<0 \quad \text{(Maximum)}$$

$$Ryy(x,y)=0-6(1)-0=-6$$

$$Ryy(x,y)=0-0-4=-4$$

And the relative maximum value at (19.5, 0.5) is given by

$$D=Rxx(19.5,0.5).Ryy(19.5,0.5)-Rx^2y(19.5,0.5)$$

$$=(-4)(-6)-(-4)^2=24-16=8$$

☆☆☆☆☆